# On the Parallelization of Spectrum Defragmentation Reconfigurations in Elastic Optical Networks

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***Abstract*—Flexible-grid elastic optical networks (EONs) have attracted intensive research interests for the agile spectrum management in the optical layer. Meanwhile, due to the relatively small spectrum allocation granularity, spectrum fragmentation has been commonly recognized as one of the key factors that can deteriorate the performance of EONs. To alleviate spectrum fragmentation, various defragmentation (DF) schemes have been considered to consolidate spectrum utilization in EONs through connection reconfigurations. However, most of the previous approaches operate in the sequential manner (Seq-DF), i.e., involving a sequence of reconfigurations to progressively migrate highly fragmented spectrum utilization to consolidated state. In this paper, we propose to perform the DF operations in a parallel manner (Par-DF), i.e., conducting all the DF-related connection reconfigurations simultaneously. We first provide a detailed analysis on the latency and disruption of Seq-DF and Par-DF in EONs, and highlight the benefits of Par-DF. Then, we study two types of Par-DF approaches in EONs, i.e., reactive Par-DF (re-Par-DF) and proactive Par-DF (pro-Par-DF). We perform hardness analysis on them, and prove that the problem of re-Par-DF is $\mathcal{NP}$-hard in the strong sense while pro-Par-DF is an $\mathcal{APX}$-hard problem. Next, we focus on pro-Par-DF and propose a Lagrangian-relaxation (LR) based heuristic to solve it time-efficiently. The proposed algorithm decomposes the original problem into several independent subproblems and ensures that each of them can be solved efficiently. The LR based approach informs us the proximity of current feasible solution to the optimal one constantly, and offers a near-optimal performance (relative dual gap $<5\%$) within 500 iterations in most simulations. Extensive simulations also verify that the proposed pro-Par-DF approach outperforms Seq-DF in terms of the DF Latency, Disruption and Cost.**

***Index Terms*—Elastic optical networks (EONs), Lagrangian relaxation, parallel defragmentation, spectrum defragmentation.**

## I. INTRODUCTION

RECENTLY, the agile spectrum management in flexible-grid elastic optical networks (EONs) has become a hot research topic, as EONs place unprecedented intelligence in the optical layer [1]. The bandwidth-variable transponders in EONs operate on several narrow-band (12.5 GHz or less) frequency slots (FS') that are spectrally contiguous and achieve data transmission over them. Therefore, unlike the traditional fixed-grid WDM networks that allocate spectra based on discrete wavelength channels (50 or 100 GHz), EONs provide a much finer bandwidth allocation granularity. This mechanism enables EONs to provision just-enough bandwidth to each connection request dynamically and improves spectral efficiency significantly. It is known that to support emerging applications such as datacenter networks and cloud computing seamlessly, there is an increasing demand for the underlying optical networking infrastructure to become flexible and dynamic [1], [2]. To this end, EONs are promising and desirable.

EONs also bring new challenges to network planning and provisioning. For instance, in order to serve a connection request, the control plane needs to manipulate blocks of contiguous FS' instead of independent wavelength channels [3]–[6]. More importantly, due to the small spectrum granularity, flexible-grid wavelength plan [7] and dynamic lightpath management, the problem of spectrum fragmentation in EONs will be much more severe than that in WDM networks [8]. Basically, spectrum fragmentation refers to the existing of non-aligned, isolated, and small-sized unused FS blocks in optical spectra due to dynamic lightpath operations [8], [9]. As these FS blocks can hardly be used for future requests, they cause low spectrum utilization and increase the blocking probability of requests with relatively large bandwidth requirements [8].

In principle, the spectrum fragmentation in EONs is similar to the resource fragmentation in other systems, such as computer storage and synchronous optical network (SONET) systems, for which various defragmentation (DF) approaches have been proposed previously [10], [11]. Nevertheless, the spectrum fragmentation in EONs also possesses some unique characteristics, which make the DF approaches developed for other systems not applicable. For example, due to the spectrum contiguity and continuity constraints, optical spectra on different fibers in an EON cannot be defragmented independently [12]. Therefore, DF in EONs becomes a new and challenging problem that requires immediate attentions.

Previously, researchers have proposed a few DF algorithms [9], [13]–[19], which relied on connection reconfigurations to reduce spectrum fragmentation in EONs. However, most of them operated in a sequential manner, i.e., involving a sequence of connection reconfigurations (path rerouting and/or spectrum retuning) to progressively migrate the optical spectra in EONs from highly-fragmented to consolidated. It is known that sequential DF (Seq-DF) suffers from several drawbacks. Firstly, Seq-DF can lead to long traffic disruption. As we will show in Section IV-C, the disruption duration caused by Seq-DF might be in the order of seconds. Another drawback

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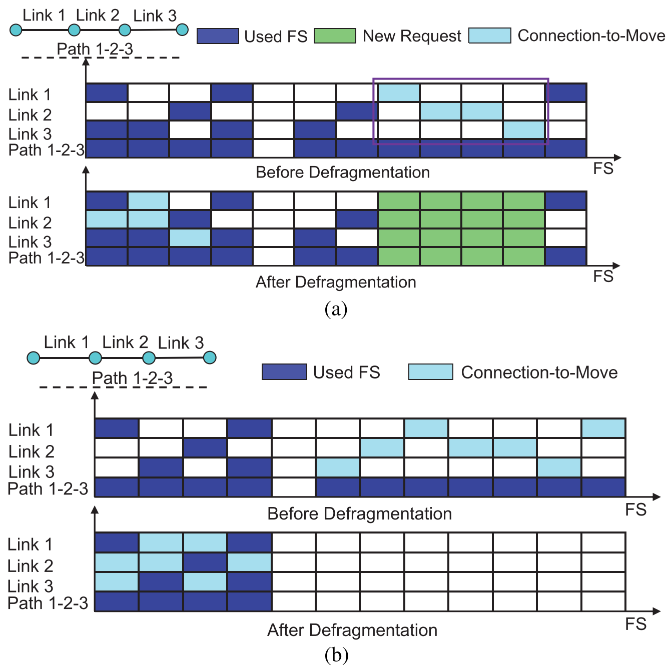


Fig. 1. Examples of parallel defragmentation (Par-DF) schemes. (a) Reactive parallel defragmentation (re-Par-DF). (b) Proactive parallel defragmentation (pro-Par-DF).

is that Seq-DF usually involves complicated traffic migration procedure [20]. In light of these issues, we need to investigate how to realize the parallelization of DF operations in EONs.

From the operation point of view, parallel DF (Par-DF) can be conducted in either reactive or proactive manner. Reactive Par-DF (re-Par-DF) scheme reconfigures certain existing connections to accommodate a new request that would be blocked otherwise. Fig. 1(a) shows an example of re-Par-DF. Here, we need to serve a new request for 4 FS' to traverse *Links* 1–3, each of which can accommodate 12 FS' at most. Before the DF, even though there are sufficient FS' on each individual link, the request would still be blocked because the available FS' are not aligned along the links to satisfy the spectrum contiguity and continuity constraints. The DF reconfigures connections to release *FS*' 8–11 on the path for the request.

Proactive Par-DF (pro-Par-DF) monitors network status proactively and consolidates network-wide spectrum utilization when necessary. The DF in Fig. 1(b) reconfigures all the existing connections that use *FS*' 6–12 on *Links* 1–3. After the DF, the network-wide spectrum utilization is less fragmented and the EON can accommodate more future requests. As correlated connections can compete for FS' during pro-Par-DF and reconfiguring one unwisely can make several others unmovable, designing an effective pro-Par-DF algorithm is challenging. The hardness of solving pro-Par-DF is still unknown, and the efficient algorithms that can be implemented in dynamic EONs are under-explored.

In this paper, we propose and design several new Par-DF schemes for EONs. The schemes can eliminate the dependency among connection reconfigurations and hence enable the connections to be migrated simultaneously. Our theoretical analysis and simulation results indicate that the proposed Par-DF can provide short traffic disruption and DF latency, simple traffic migration procedure, and competitive request blocking reduction. The contributions of this work are as follows.

- To the best of our knowledge, this is the first work to provide the hardness analysis for the problems of re-Par-DF and pro-Par-DF in EONs.
- We prove that re-Par-DF in EONs is a strongly $\mathcal{NP}$-hard problem, even though the similar problem in fixed-grid WDM networks can be solved in polynomial time [21].
- We formulate a compact integer linear programming (ILP) model to solve pro-Par-DF in EONs, and prove that the problem's hardness is $\mathcal{APX}$-hard.
- We develop a Lagrangian relaxation approach to decompose the ILP model of pro-Par-DF into independent subproblems that can all be solved quickly. Through extensive simulations, we demonstrate that the Lagrangian relaxation-based algorithm can offer near-optimal solutions within a reasonable number of iterations ($\leq$500).
- Under the background of softward-defined EON, we provide detailed analysis and comparison of Par-DF and Seq-DF, in terms of DF latency and traffic disruption.

The rest of the paper is organized as follows. Section II summarizes the related work. In Section III, we introduce EONs, discuss the network architecture and enabling techniques, and formulate the network model. The problems of Seq-DF and Par-DF are described in Section IV, and the DF latency and traffic disruption caused by them are also analyzed. Section V studies re-Par-DF in detail and analyzes its intractability. Section VI provides the ILP formulation for pro-Par-DF and analyzes its hardness, and the Lagrangian relaxation-based pro-Par-DF algorithm is discussed in Section VII. Section VIII presents the simulation results for performance evaluation. Finally, Section IX summarizes the paper.

## II. Related Work

To relieve spectrum fragmentation in EONs, people have investigated both preemptive and responsive approaches. The preemptive approaches try to minimize spectrum fragmentation at the time when each connection request is originally served [22]–[24]. Wang *et al.* proposed a spectrum management method in [22], which partitioned the spectrum resources into multiple regions to serve connections with different bandwidth requirements. Their simulation results showed that by doing so, the network's fragmentation-ratio and blocking probability could be reduced. The study in [23] suggested that by leveraging routing and spectrum assignment (RSA) with multipath routing, spectrum fragmentation could be relieved. In [24], we proposed fragmentation-aware RSA algorithms based on the fragmentation-related metrics to reduce blocking probability.

The preemptive approaches can relieve spectrum fragmentation in EONs, but they are far from enough, as fragmentation still exists and can deteriorate network performance even when they have already been applied [19]. Therefore, researchers turned to the responsive approaches that utilize network reconfigurations to reduce spectrum fragmentation [9], [16]–[19], [21]. Since the working principle is similar to that of storage defragmentation in computer systems, people tend to refer to them as "spectrum defragmentation" (DF). In [16], Castro *et al.* formulated an ILP model to solve the problem of reactive sequential DF and also proposed several meta-heuristic algorithms. Aiming at reducing the path setup delay, they only used spectrum retuning in each reconfiguration. Based on the push-pull based spectrum retuning technique, a DF algorithm was developed in [17]. Even though the algorithm could reduce blocking probability effectively, it still utilized the sequential DF and could not avoid the complicated traffic migration and long latency in each reconfiguration. In [9], Yin *et al.* proposed a maximum-independent-set based algorithm to tackle the reactive parallel DF problem in EONs and leveraged all-optical spectrum conversion to demonstrate the DF scheme experimentally. However, due to the complexity of the proposed algorithm, the DF scheme has scalability issues and can hardly be applied to large-scale networks. On the other hand, the authors of [18] also developed a maximum-independent-set based algorithm for sequential DF, in which they built an auxiliary graph based on the network spectrum utilization to determine the traffic migration sequence. In [19], we studied adaptive DF for the EONs with time-varying traffic and proposed several algorithms for efficient traffic migration.

Regarding the parallel reconfiguration in optical networks, the authors of [21] have proposed a polynomial-time algorithm to solve the problem in fixed-grid WDM networks. However, the analysis on problem hardness, reconfiguration latency, and traffic disruption are not directly applicable to EONs. From the discussion above, we can see that most of the previous studies in this area were targeted for sequential DF (Seq-DF), while to the best of our knowledge, the problem of parallel DF (Par-DF) in EONs is still under-explored.

## III. Flexible-Grid Elastic Optical Networks

### A. Network Architecture

Due to the huge amount of bandwidth provided by optical fibers, optical networks are indispensable in today's Internet infrastructure. Currently, the optical transport networks are implemented with fixed-grid WDM systems, which suffer from coarse spectrum allocation granularity, low spectrum efficiency as well as limited flexibility in the optical layer. In order to address these issues, one needs EONs equipped with bandwidth-variable transponders (BV-T) and switches (BV-WSS) that can allocate just-enough bandwidth to various traffic demands (as shown in Fig. 2(a)). The major difference between EONs and the conventional WDM networks is that EONs can support various requests for low and ultra-high bit-rates adaptively with sub-wavelength FS' and super-channels [4], [25]. In order to realize this, EONs operate according to a flexible-grid wavelength plan [7]. Fig. 2(b) compares the flexible-grid wavelength plan with the fixed-grid one.

### B. Key Components and Enabling Techniques

Three things are essential for EONs, namely, BV-Ts, BV-WSS', and an intelligent control plane. While for DF in EONs, we need key enabling technologies such as the path rerouting and/or spectrum retuning.

*1) Bandwidth-Variable Transponder (BV-T):* By leveraging the multi-carrier optical transmission techniques such as CO-OFDM and Nyquist-WDM, people have realized BV-Ts that groom the capacities of several spectrally-contiguous FS' and operate on the flexible-grid wavelength plan. When the

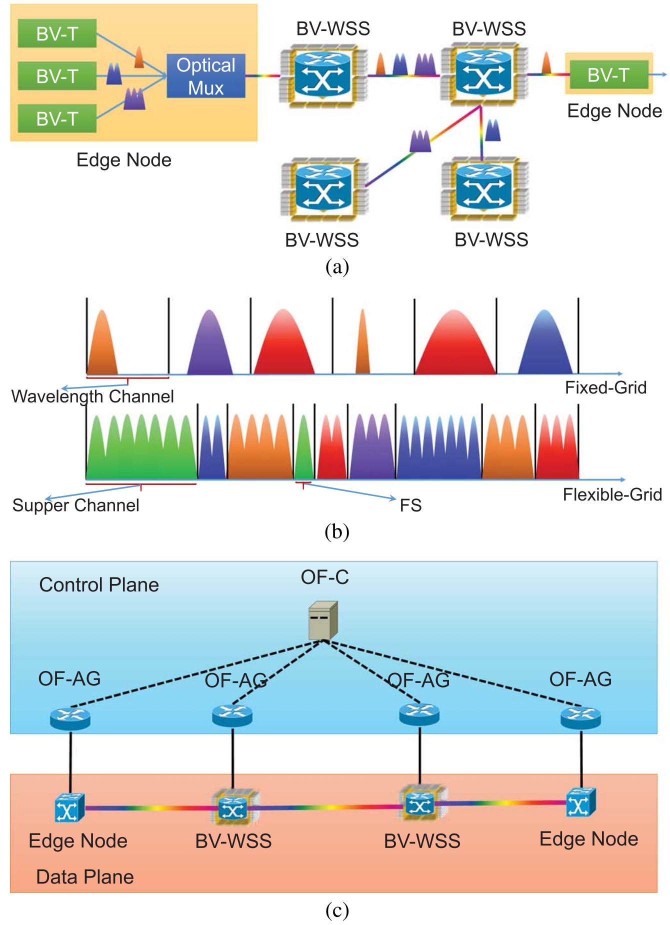


Fig. 2. Operation principle of flexible-grid EONs. (a) Network architecture of EON, BV-T: Bandwidth-variable transponder, BV-WSS: Bandwidth-variable wavelength-selective switch, Optical Mux: Optical spectrum multiplexer. (b) Illustrations of fixed-grid and flexible-grid wavelength plans. (c) Centralized network control and management (NC&M) in SD-EON. OF-AG: OpenFlow agent, OF-C: OpenFlow controller.

traffic demand changes, the BV-T can adjust the allocated bandwidth by assigning a different number of FS'.

*2) Bandwidth-Variable Wavelength-Selective Switch (BV-WSS):* A BV-WSS allows flexible spectra to be switched correctly from its input to output ports. Thanks to the technology advances on the liquid crystal-on-silicon (LCOS), an LCOS-WSS can achieve the switching granularity at 6.25 GHz or less successfully [26].

*3) Intelligent Control Plane:* The flexible nature of EONs makes the intelligent control plane a must-have component for cost-effective resource management. Recently, the software-defined network (SDN) architecture has been demonstrated for efficient network control and management (NC&M) [27]–[30]. The centralized control plane provided by SDN opens up the possibility for efficient DF in EONs, as it can collect the information of network-wide spectrum utilization and perform re-optimization globally. Fig. 2(c) shows the network architecture of SD-EON [27]. The data plane is built with BV-WSS' and the edge nodes that contain BV-Ts, while the control plane consists of a centralized OpenFlow controller (OF-C) and several OpenFlow agents (OF-AGs), each of which attaches to a network element in the data plane.

*4) Reconfiguration Techniques:* In the DF operation, an EON relies on path rerouting and/or spectrum retuning to accomplish connection reconfiguration. Path rerouting means that the reconfiguration changes both the routing path and the spectrum assignment of a connection, i.e., Re-RSA. People have proposed several make-before-break techniques to facilitate hitless path rerouting [20]. On the other hand, with spectrum retuning, we can also realize connection reconfiguration. Specifically, it only involves reassignment of the spectrum allocation, while the routing path of the connection stays unchanged. Previously, a push-pull based spectrum retuning technique has been proposed and experimentally demonstrated in [31]. Nevertheless, since it only allows a connection to be shifted over unused and contiguous spectra, the push-pull based technique limits the flexibility of DF [32]. Moreover, the push-pull based technique does not support Par-DF, since the reconfiguration with it has to go through multiple steps. Meanwhile, Proietti *et al.* demonstrated a hop-tuning based spectrum retuning technique, which could support full spectrum-hopping within a relatively short duration ($\leq 1\ \mu$s) [32]. Apparently, the hop-tuning based technique fits well with the requirement of Par-DF, and our DF algorithms will be based on it.

It is known that path rerouting provides a relatively large solution space for the connection reconfiguration and hence can make the DF operation more effective. However, this also makes the computation for DF schemes more complex [16]. More importantly, with path rerouting, the connection reconfiguration needs to adjust more BV-WSS' in each DF operation, and in the optical layer, rerouting may involve optical power re-equalization due to the deletion and insertion of connections on certain links [31]. Hence, we only consider spectrum retuning for DF in this work.

### C. Network Model for Spectrum Defragmentation

We use the following notations to model the DF in EONs.

- $G(V, E)$: Physical topology, where $V$ and $E$ are the sets of switching nodes and fiber links, respectively.
- $F$: Number of FS' on each link $e \in E$.
- $b_e$: The $F$-bit bit-mask to represent the spectrum utilization on link $e$. If $b_e[j] = 1$, the $j$-th FS on $e$ is used and unavailable, otherwise, $b_e[j] = 0$.
- $p_{s,d}^{(k)}$: The $k$-th routing path from $s$ to $d$ in $G(V, E)$.
- $\mathbb{P}_{s,d}$: The set of $K$ paths from $s$ to $d$ in $G(V, E)$.
- $C_i(s_i, d_i, n_i)$: The $i$-th connection in the EON, where $i$ is the unique index, $s_i$ and $d_i$ are the source and destination nodes, and $n_i$ is the number of FS' that it uses.
- $p_i$: The routing path that carries $C_i$, and $p_i \in \mathbb{P}_{s_i,d_i}$.
- $a_i$: The $F$-bit bit-mask to represent the spectrum assignment of $C_i$ along path $p_i$. If $a_i[j] = 1$, the $j$-th FS is allocated to $C_i$, otherwise, $a_i[j] = 0$.

Initially, a connection is set up with RSA, i.e., selecting $p_i$ from $\mathbb{P}_{s_i,d_i}$ and determining $a_i$. In this work, we assume that there is no spectrum converter in the EON and each connection is provisioned all-optically end-to-end for energy-saving [33], [34]. Hence, the spectrum assignment stays the same for all the links on $p_i$, i.e., satisfying the spectrum continuity constraint. As we denote the spectrum assignment on $p_i$ with $a_i$, the spectrum continuity constraint is taken care of automatically. There are two additional constraints for the RSA to obey, and a request will be blocked, if no feasible RSA solution can be found based on the current network status under them.

- *Spectrum Non-overlapping Constraint*: the FS' assigned to $C_i$ cannot be used by other connection(s), i.e.,

$$\sum_{j=1}^{F} a_i[j] \cdot b_e[j] = 0, \quad \forall e \in p_i. \tag{1}$$

- *Spectrum Contiguity Constraint*: all the assigned FS' have to be spectrally contiguous to form an FS-block, i.e.,

$$\sum_{j=1}^{F} a_i[j] \cdot ROR(a_i, 1)[j] = \begin{cases} n_i - 1, & n_i \in [1, F), \\ F, & n_i = F, \end{cases} \tag{2}$$

where $ROR(a_i, 1)$ is also an $F$-bit bit-mask, which is obtained by circular-right-shifting $a_i$ for one bit.

Spectrum DF reconfigures certain existing connections with the objective of consolidating the spectrum utilization in the EON. Specifically, if connection $C_i$ needs to be reconfigured, we need to first accomplish an one-to-one mapping,

$$M : \{p_i, a_i\} \mapsto \{p'_i, a'_i\}, \tag{3}$$

where $\{p'_i, a'_i\}$ denotes the new RSA for the reconfiguration, i.e., $p'_i$ is the new routing path and $a'_i$ is the new spectrum assignment. Since we only consider spectrum retuning in this work, we have $p_i = p'_i$ and the mapping becomes,

$$M_{FS} : a_i \mapsto a'_i. \tag{4}$$

If $M_{FS}$ is not feasible for $C_i$, the connection is unmovable. With $M_{FS}$ determined for each connection, DF invokes the traffic migration to reconfigure all the movable connections. Apparently, the mapping and migration are correlated such that $M_{FS}$ determines the migration scheme, i.e., whether it can be done in the sequential (Seq-DF) or parallel (Par-DF) manner.

One important concept regarding Seq-DF and Par-DF is the dependency between the connections [20].

*Definition 1:* For two connections $C_i$ and $C_j$ $(i \neq j)$, if the new RSA of $C_i$ will use the FS' that are currently occupied by $C_j$, i.e., "$p'_i \cap p_j \neq \emptyset$" and "$\sum_{k=1}^{F} a'_i[k] \cdot a_j[k] \neq 0$" are both satisfied, we say that $C_i$ **depends on** $C_j$, denoted as $C_j \Leftarrow C_i$.

Obviously, if $C_j \Leftarrow C_i$, the migration for $M_{FS}(C_i)$ cannot be done until the one for $M_{FS}(C_j)$ has been done, which leads to Seq-DF, otherwise, Par-DF is feasible.

### D. Performance Metrics

For all the connection requests $\{C_i\}$ that arrive within $[0, T]$, we use $N(T) = \sum_i n_i$ and $N_b(T) = \sum_{i,\ blocked} n_i$ to denote the total requested and blocked bandwidth, respectively.

*Definition 2:* The **Bandwidth Blocking Probability (BBP)** of service provisioning is defined as

$$BBP = \lim_{T \to \infty} \frac{N_b(T)}{N(T)}, \tag{5}$$

which is an important metric to evaluate network performance.

*Definition 3:* The **Network Utilization** is defined as the average spectrum utilization per fiber link, which can describe an algorithm's effectiveness on serving lightpath requests.

*Definition 4:* The **DF Disruption** of $C_i$ is defined as the time duration for accomplishing the migration for $a_i \mapsto a'_i$. Note that in this duration, the BV-T of $C_i$ has to pause the data transmission to avoid data loss.

*Definition 5:* The **DF Latency** is defined as the time duration for completing all the necessary connection reconfigurations in a DF operation.

*Definition 6:* The **DF Cost** is defined as the number of connection reconfigurations, similar to that defined for the resource DF in SONET systems [11].

## IV. Spectrum Defragmentation With Connection Reconfigurations

### A. Sequential Defragmentation (Seq-DF)

In Seq-DF, to reduce the DF Disruption of each movable connection that needs to be reconfigured, we need to design the migration sequence carefully and try to apply the "move-to-vacancy (MTV)" scheme [20] to as many connections as possible. Specifically, for any movable connection $C_i$, MTV requires that $M_{FS}$ must return new spectral location $a_i'$ that is currently unoccupied. Hence, we can reconfigure the connection by setting up the new lightpath with unused spectra first and then tuning its BV-T's from the original spectrum assignment to the new one. By doing so, we make sure that the connection only experiences the disruption from BV-T retuning and avoids the disruption from BV-WSS reconfiguration.

The migration sequence of Seq-DF can be determined with the aid of an auxiliary graph for dependency [19], [20], $G^d(V^d, E^d)$, where $V^d$ and $E^d$ are the node and link sets, respectively. Each node $v^d \in V^d$ is for a movable connection selected by the DF. If $C_j \Leftarrow C_i$ and $v_j^d$ and $v_i^d$ are their corresponding nodes in $G^d(V^d, E^d)$, there is a directed link $v_i^d \rightarrow v_j^d$ in $G^d(V^d, E^d)$.

Fig. 3(a) shows an intuitive example of Seq-DF, which involves reconfiguring three connections $C_1$, $C_2$ and $C_3$. Based on the spectrum utilization before DF, we find $C_1 \Leftarrow C_3$ and $C_2 \Leftarrow C_3$, as shown in the auxiliary graph for dependency. Since there is no dependency between $C_1$ and $C_2$, they can be migrated with MTV simultaneously. Then, after this batch of reconfigurations, $C_3$ can be migrated. For more general cases, the migration sequence of Seq-DF can be obtained by running a revised topological sorting algorithm on $G^d(V^d, E^d)$. The topological sorting divides the connections into different groups, and Seq-DF can reconfigure all the connections in a group simultaneously as one step. For instance, in Fig. 3(a), $C_1$ and $C_2$ belong to the group for *Step* 1. More detailed analysis on Seq-DF can be found in [19].

### B. Parallel Defragmentation (Par-DF)

In Par-DF, the migration procedure is much simpler, and all the movable connections can be reconfigured simultaneously in only one batch. In order to achieve this, we have to make sure that there is no dependency among the movable connections, by properly designing the mapping $M_{FS}$ for them. Note that here, we have to enforce the MTV scheme for connection reconfiguration. Fig. 3(b) illustrates an instance of Par-DF, where $C_1$, $C_2$ and $C_3$ can be migrated in parallel.

### C. Analysis on DF Disruption and Latency

Next, we assume that the DF schemes are implemented in the SD-EON architecture in Fig. 2(c), and analyze the DF Disruption and Latency of Par-DF and Seq-DF.

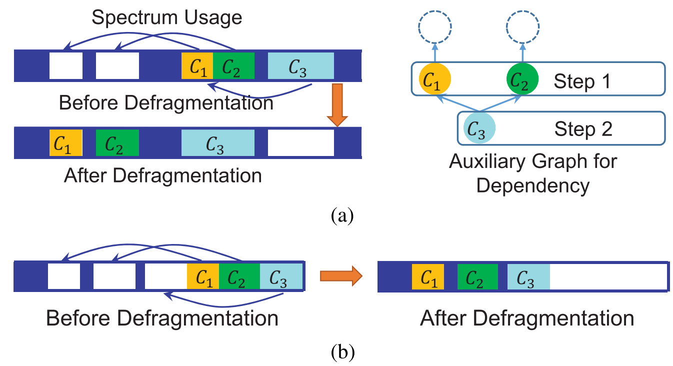


Fig. 3. Seq-DF and Par-DF examples. (a) Example of sequential defragmentation (Seq-DF). (b) Example of parallel defragmentation (Par-DF).

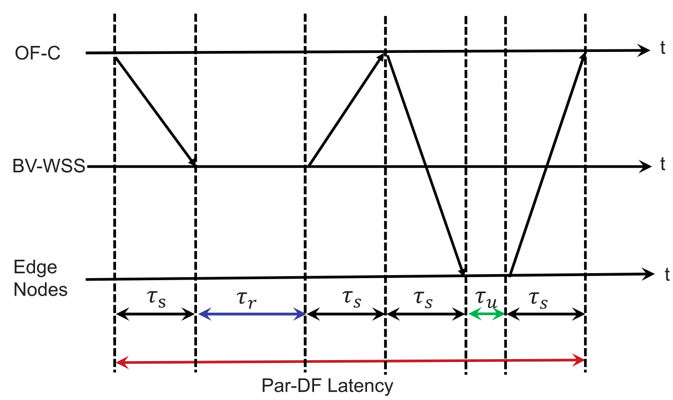


Fig. 4. Procedure of reconfiguring one connection in Par-DF.

*1) Par-DF Case:* Fig. 4 shows the procedure of reconfiguring one connection. Firstly, OF-C sends messages to the relate OF-AGs and instructs them to reconfigure the BV-WSS'. The signaling delay $\tau_s$ from OF-C to OF-AGs can be calculated as the summation of transmission delay, propagation delay and queuing delay, and it is in the order of milliseconds [35]. The time duration for the BV-WSS' to reconfigure is denoted as $\tau_r$, which is also in the order of millisecond, according to [36]. Then, the OF-AGs on the BV-WSS' send acknowledgments to OF-C. When OF-C receives the acknowledgements from all the related OF-AGs, it sends messages to the source and destination nodes and makes their BV-Ts retune to the new FS'. According to the experimental results in [32], the duration $\tau_u$ for a BV-T to accomplish retuning is in the order of microseconds if it uses the hop-retuning technique. When the retuning on the BV-Ts is completed, the source and destination nodes send acknowledgments to OF-C, which concludes the connection reconfiguration.

It can be seen that for the aforementioned scheme, traffic disruption only happens when the BV-Ts are performing the spectrum retuning, and hence, the DF Disruption of Par-DF, $D_P$, can be estimated as,

$$D_P = \tau_u. \tag{6}$$

Meanwhile, since all the movable connections are reconfigured in parallel in Par-DF, the DF Latency of Par-DF, $L_P$ is

$$L_P = \max_i \left(4 \cdot \tau_{s,i} + \tau_{r,i} + \tau_{u,i}\right), \tag{7}$$

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where $i$ is the index of a movable connection $C_i$, and $\tau_{s,i}$, $\tau_{r,i}$ and $\tau_{u,i}$ are the corresponding delay and durations during reconfiguring $C_i$. Since the routing path that carries $C_i$ can be a multi-hop one, the signaling delays from OF-C to different OF-AGs and the reconfiguration delays on different BV-WSS' can be different. Hence, we take the maximum values of them as $\tau_{s,i}$ and $\tau_{r,i}$ when calculating $L_P$. With (6), (7), we can determine that for the Par-DF case, $D_P$ and $L_P$ are in the orders of microseconds and milliseconds, respectively.

*2) Seq-DF Case:* As the discussion in Section IV-A points out, the connection reconfigurations in Seq-DF are in batches. And reconfiguring a group of connections in one step is equivalent to performing one parallel reconfiguration. Hence, the DF Latency of Seq-DF, $L_S$, can be estimated as,

$$L_S \approx M_b \cdot L_P, \tag{8}$$

where $M_b$ is the number of steps in Seq-DF. With the Seq-DF scenario discussed in Section IV-A, we need to perform BV-WSS reconfiguration in every step for applying the MTV scheme in the best-effort way. Note that this is for the worst case, and in certain special cases, we do not need to reconfigure the BV-WSS' in every step. For instance, Seq-DF needs to reconfigure three connections, $C_1$, $C_2$ and $C_3$, and we have $C_1 \Leftarrow C_2 \Leftarrow C_3$. If the connections use the same routing path and occupy the same number of FS', then in the second step, we do not need to reconfigure the BV-WSS' for migrating $C_2$ since $C_2$ will use the FS' that are originally assigned to $C_1$, which have already been configured in the BV-WSS'. Hence, we just tune the BV-Ts of $C_2$ to the new spectral location and the traffic migration is accomplished. However, in general cases, the connections that use different routing paths can have dependencies if their routing paths share certain link(s) or they may occupy different numbers of FS'. Therefore, Seq-DF has to reconfigure the BV-WSS' in every step, which is the case we consider in the following analysis.

Then, we analyze the DF Disruption of Seq-DF, for which we need to consider two scenarios, i.e., the auxiliary graph for dependency, $G^d(V^d, E^d)$, is acyclic or cyclic.

- $G^d(V^d, E^d)$ is acyclic, e.g., the one shown in Fig. 3(a). This means that no movable connections need to be suspended[1] until others have been reconfigured, and then its DF Disruption, $D_S^{ac}$, equals to that in Par-DF, i.e.,

$$D_S^{ac} = D_P. \tag{9}$$

- $G^d(V^d, E^d)$ is cyclic. This means that a connection needs to be suspended to make room for the others. For instance, in Fig. 5(a), as we have $C_1 \Leftarrow C_2$, $C_2 \Leftarrow C_3$ and $C_3 \Leftarrow C_1$, Seq-DF has to suspend $C_2$ in the cycle to make sure that $C_1$ and $C_3$ can be migrated with MTV. $C_2$ will be resumed when $C_1$ and $C_3$ have been reconfigured, as shown in Fig. 5(b). Then, we consider the worst case, and the DF Disruption $D_S^c$ can be estimated as,

$$D_S^c \approx M_c \cdot L_P, \tag{10}$$

where $M_c$ is the hop-count of the longest cycle.

[1]During network reconfiguration, we may need to suspend a connection to avoid data loss, i.e., its service is temporarily interrupted as the BV-Ts stop to transmit/receive data.

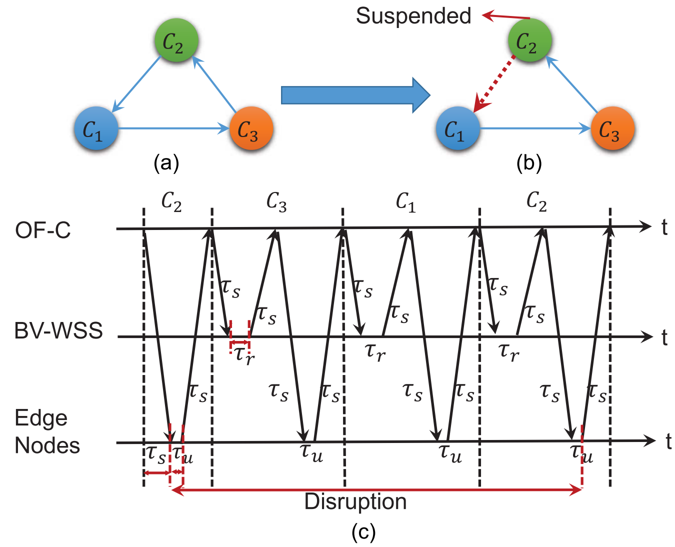


Fig. 5. Example of Seq-DF procedure for a cyclic $G^d(V^d, E^d)$.

More specifically, (10) can be explained with the procedure in Fig. 5(c). The reconfiguration procedure is as follows. Firstly, for suspending $C_2$, OF-C informs its edge nodes to stop the data transmission in the BV-Ts, which takes $2 \cdot \tau_s + \tau_u$. The service of $C_2$ becomes stalled when the BV-Ts start to switch off, and hence within the first slot in Fig. 5(c), the disruption of $C_2$ is $\tau_s + \tau_u$. Then, we migrate $C_1$ and $C_3$ with MTV, and within the second and third slots, the service of $C_2$ keeps being stalled. In the fourth slot, we resume $C_2$ and its service becomes active after the BV-Ts being tuned to the new FS'. Finally, we can calculate the DF Disruption of $C_2$ as $D_S^c = 3 \cdot (4 \cdot \tau_s + \tau_r + \tau_u) + \tau_u = 3 \cdot L_P + \tau_u$. As we usually have $L_P \gg \tau_u$ and $M_c = 3$ in this example, the analysis shows $D_S^c \approx M_c \cdot L_P$.

Eqs. (8)–(10) suggest that Par-DF causes much shorter DF Latency and Disruption than Seq-DF, and the corresponding reductions are related with $M_b$, $M_c$ and $\tau_s$.

## V. Reactive Parallel Defragmentation

In this work, re-Par-DF refers to the Par-DF scheme where DF is performed for accommodating a request that would be blocked otherwise. Basically, we evacuate an FS-block on one of the request's candidate routing paths to fit it in, as in Fig. 1(a). This section analyzes the hardness of solving re-Par-DF in EONs, by leveraging some of the conclusions in [10].

### A. Problem Definition

*Definition 7:* Given an EON $G(V, E)$, a set of existing connections $\mathbb{C} = \{C_i\}$, and a pending connection $C_j(s_j, d_j, n_j)$ ($C_j \notin \mathbb{C}$) that would be the blocked without DF, **the problem of re-Par-DF** is that whether it is possible to form an FS-block consisting of $n_j$ contiguous available FS' on any of the paths in $\mathbb{P}_{s_j,d_j}$, by retuning certain connections in $\mathbb{C}$ with MTV.

Next, we prove the re-Par-DF problem is $\mathcal{NP}$-hard in the strong sense, which is inspired by the work in [10].

### B. Intractability Analysis

*Theorem 1:* The problem of re-Par-DF is $\mathcal{NP}$-hard in the strong sense.

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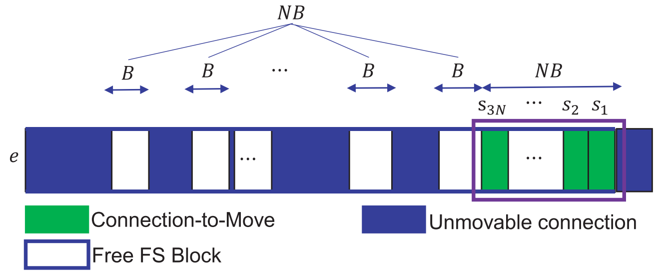
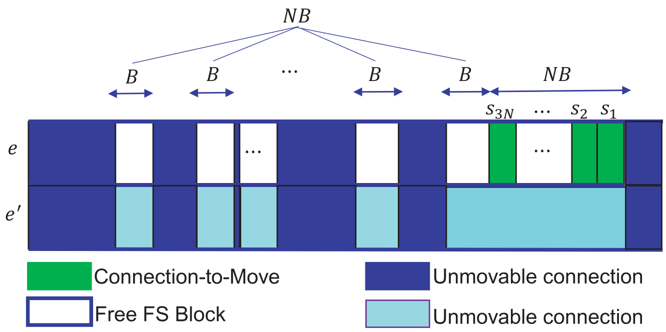



(a)




(b)

Fig. 6. Network instances for intractability analysis of re-Par-DF. (a) Network instance to prove re-Par-DF is not in the $\mathcal{NP}$ class. (b) Network instance to prove re-Par-DF is strongly $\mathcal{NP}$-hard.

*Proof:* In order to prove the $\mathcal{NP}$-completeness of re-Par-DF, we reduce the famous 3-partition problem [37] to a related re-Par-DF. The 3-partition problem is to decide whether a given set of positive integers can be partitioned into disjoint subsets that all have the same summations, and this problem is know to be $\mathcal{NP}$-complete in the strong sense [37]. More precisely, given a set $S$ that contains $3N$ positive integers, can $S$ be partitioned into $N$ disjoint subsets $S_1, S_2, \cdots, S_N$ such that $\sum_{s_i \in S_1} s_i = \sum_{s_i \in S_2} s_i = \cdots = \sum_{s_i \in S_N} s_i = B$, with $B \in Z^+$ and $s_i \in [(B/4),(B/2)]$, $\forall s_i \in S$? Here, $s_i$ refers to an integer in $S$ or any of its subsets, and we have $\sum_{s_i \in S} s_i = N \cdot B$.

We first prove that the problem of re-Par-DF is not in the $\mathcal{NP}$ class. If the pending connection $C_j$ can only use the path that contains link $e$, whose spectrum utilization is shown in Fig. 6(a), we can see that there are $N$ vacant FS-blocks $I_1, \cdots, I_N$ whose sizes are all $B$ FS'. We assume that the bandwidth requirement of $C_j$ equals $N \cdot B$ FS', i.e., $n_j = N \cdot B$. Since the connections outside of the green FS-block (i.e., the blue FS-blocks in Fig. 6(a)) are all unmovable, we cannot retune them to accommodate $C_j$, otherwise, there will be dependency among the connections and Par-DF is not feasible. Hence, in order to evacuate the green FS-block for $C_j$, as shown in Fig. 6(a), we guess the solution of re-Par-DF as to retune the connections that occupy the FS' within the green FS-block (whose size is $N \cdot B$ FS') using MTV. Suppose there are $3N$ connections in the green FS-block, each of which occupies a bandwidth within $[(B/4),(B/2)]$ FS'. We can see that each instance of 3-partition problem can create an instance of re-Par-DF as we constructed above. Then, to give an answer to the guess of re-Par-DF in Fig. 6(a) will imply a solution to an instance of the 3-partition problem, which is strongly $\mathcal{NP}$-complete. We thus cannot give an answer to any guess of re-Par-DF in polynomial time, which proves that the problem of re-Par-DF is not in the $\mathcal{NP}$ class.

Next, we prove that the problem of re-Par-DF is strongly $\mathcal{NP}$-hard. We construct another network instance as shown in Fig. 6(b), which is similar to that in Fig. 6(a) but includes an additional link $e'$. We assume that the blue FS-blocks on $e$ are used by the connections that traverse both $e$ and $e'$. Hence, they cannot be retuned in any case, because the corresponding FS-blocks on $e'$ are unavailable. Then, the connections within the green FS-block in Fig. 6(b) are the only ones that can be retuned. And each instance of 3-partition problem can create an instance like this. Hence, we prove that the general problem of re-Par-DF is $\mathcal{NP}$-hard in the strong sense. ■

Previously, the authors of [21] have proved that in fixed-grid WDM networks, re-Par-DF can be solved in polynomial time. Actually, the re-Par-DF studied in [21] is just a special case of the re-Par-DF problem analyzed above, i.e., all the connections have $n_i = 1$. Therefore, we can see that compared with its counterpart in WDM networks, the complexity of re-Par-DF in EONs increases dramatically. In dynamic network operation, if we want to incorporate re-Par-DF, we have to solve it within a reasonably short time. However, this is not possible due to the problem's intractability. Or in other words, as re-Par-DF is strongly $\mathcal{NP}$-hard, we cannot design an efficient pseudo-polynomial time algorithm to solve it. Hence, we turn to pro-Par-DF in the following sections for more practical solutions.

Note that in the intractable analysis above, we assume that $N$ and $B$ are unbounded and do not restrict the bandwidth requirement of each connection. Similar to the studies in [3], [38], we use these assumptions to generalize the problems in this work. However, we should notice that $N$ and $B$ are actually bounded due to the finite bandwidth on optical fiber.

## VI. Proactive Parallel Defragmentation

In this section, we discuss pro-Par-DF, which uses the push-to-the-wall scheme to consolidate the spectrum utilization in EONs, as shown in Fig. 1(b). Similar to the Seq-DF schemes [19], pro-Par-DF can be triggered either periodically or upon a specific event. In this work, without loss of generality, we assume that pro-Par-DF is triggered when a fixed number of connections have expired. We first formulate an integer linear programming (ILP) model for the problem and then prove that pro-Par-DF in EONs is $\mathcal{APX}$-hard.

In order to assist the discussion below, we define the concept of "spectrum channel" [16] as follows.

*Definition 8:* For a given existing connection $C_i(s_i, d_i, n_i) \in \mathbb{C}$, a **spectrum channel (SC)** refers to an available FS-block on its routing path $p_i$, which includes $n_i$ spectrally-contiguous FS'. Specifically, an SC can be represented with an $F$-bit bit-mask, similar to $b_e$ and $a_i$ defined in Section III-C. For $C_i$, $a_{i,k}$ denotes its $k$-th SC, while $A_i = \{a_{i,k}\}$ is the set of its SCs. Here, we have $a_{i,1} = a_i$, where $a_i$ is the current spectrum assignment, which means that if $C_i$ is not reconfigured in a DF operation, it keeps using $a_{i,1}$, which is its original spectrum assignment.

For a given network status, we pre-calculate SCs for all the movable connections, and the detailed procedure is in *Algorithm* 1. $\mathbb{I} = \{I_j\}$ denotes the set of available FS-blocks on $p_i$ (i.e., the routing path of $C_i$), whose sizes are no less than $n_i$. Here, $I_j$ is an $F$-bit bit-mask to represent the $j$-th available FS-block on the path, and if $I_j[m] = 0$, the $m$-th FS is available and included in the $j$-th FS-block, otherwise, $I_j[m] = 1$. Fig. 7 gives an example on how to pre-calculate SCs based on

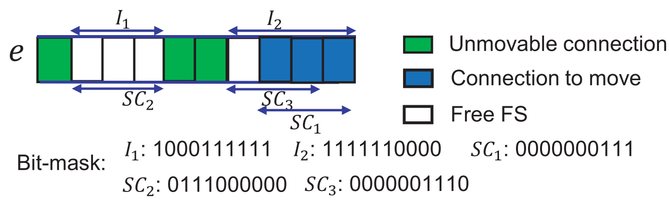



Fig. 7. Example of pre-calculating SCs and their bit-mask presentation.

the network status. As mentioned above, when calculating the SCs, we assume that the FS' that are currently occupied by $C_i$ are also available. Therefore, with the spectrum utilization, we obtain $SC_1$ and $SC_3$ as in the figure. Note that *Line* 4 indicates that we only consider the cases for pro-Par-DF, such that the spectrum assignment of each movable connection can be retuned to an SC whose start FS-index is lower than the current one. This restriction is used to enforce the push-to-the-wall scheme. The complexity of *Algorithm* 1 is $O(|\mathbb{C}| \cdot F)$.

**Algorithm 1** Pre-computation of Spectrum Channels (SCs)

1: **for** each movable connection $C_i$ in $\mathbb{C}$ **do**
2: $A_i = \{a_{i,1}\}$, $k = 2$;
3: find the first bit-1 in $a_{i,1}$ and set $l_1$ as its index;
4: obtain the set of available FS-blocks $\mathbb{I} = \{I_j\}$ on $p_i$ whose start FS-index is smaller than $l_1$ and size is not less than $n_i$;
5: **for** each FS-block $I_j$ in $\mathbb{I}$ **do**
6: find the first bit-1 in $I_j$ and set $l_2$ as its index;
7: **for** $m = l_2$ to $l_1 - 1$ **do**
8: **if** $I_j[m : m + n_i - 1] = \mathbf{0}$ **then**
9: $a_{i,k} = \mathbf{0}$;
10: $a_{i,k}[m : m + n_i - 1] = \mathbf{1}$;
11: $A_i = A_i \cup \{a_{i,k}\}$, $k = k + 1$;
12: **else**
13: **break**;
14: **end if**
15: **end for**
16: **end for**
17: **end for**

### A. Integer Linear Programming (ILP) Formulation

We formulate an integer linear programming (ILP) model based on the pre-calculated SCs to solve pro-Par-DF.

**Input Parameters**:

- $\mathbb{C}$: set of movable connections.
- $\mathbb{C}_e$: set of movable connections whose current routing paths include link $e \in E$ ($\mathbb{C}_e \subset \mathbb{C}$).
- $n_i$: the number of FS' that connection $C_i$ uses.
- $w_{i,k}$: the weight of *SC* $a_{i,k} \in A_i$, as the spectral distance from the current spectral location of $C_i$ (i.e., $a_{i,1}$) to the new one $a_{i,k}$, in terms of the number of FS'.

**Decision Variables**:

- $x_{i,k}$: Boolean variable that equals 1 if we decide to move connection $C_i$ to *SC* $a_{i,k}$, and 0 otherwise.

**Objective Function**:

$$Maximize \quad W = \sum_{i=1}^{|\mathbb{C}|} \sum_{k=1}^{|A_i|} w_{i,k} \cdot x_{i,k}, \tag{11}$$

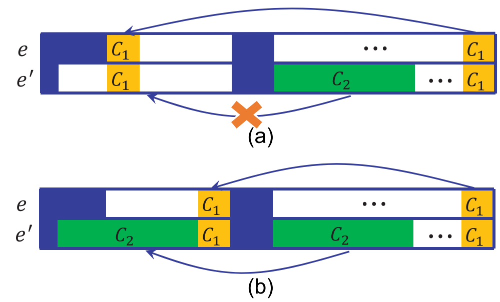



Fig. 8. Examples of (a) improper and (b) proper selections of new SCs in pro-Par-DF.

The objective function in (11) is the total weight of the SCs selected for all the movable connections. Since an SC's weight is the spectral distance between it and the corresponding connection's current spectral location, maximizing the summation of the weights can help to consolidate the spectrum usage to the lower end.

**Constraints**:

$$\sum_{k=1}^{|A_i|} x_{i,k} = 1, \quad \forall i. \tag{12}$$

Eq. (12) ensures that each movable connection still occupies an SC after the reconfiguration.

$$\sum_{\{i: C_i \in \mathbb{C}_e\}} \sum_{k=1}^{|A_i|} a_{i,k}[j] \cdot x_{i,k} \leq 1, \quad \forall j, \quad \forall e \in E. \tag{13}$$

Eq. (13) ensures that any FS on a link $e \in E$ can only be allocated to one connection.

$$i, j, k \in Z^+, i \in [1, |\mathbb{C}|], k \in \left[1, \max_i(|A_i|)\right], j \in [1, F]. \tag{14}$$

Eq. (14) restricts the ranges of the indices.

The numbers of constraints and variables in the ILP are $|\mathbb{C}| + |E| \cdot F$ and $\sum_i |A_i|$, respectively. Hence, the optimization problem is a large-scale one. For instance, suppose that we have 100 movable connections (i.e., $|\mathbb{C}| = 100$), the number of fiber links in the EON is $|E| = 44$ (e.g., the NSFNET topology in [4]), each fiber can accommodate $F = 358$ FS', and the average number of SCs for each connection is 10. Then, the problem has $\sim 10^4$ constraints and $\sim 10^3$ variables.

### B. Intractability Analysis

Since correlated connections can compete for spectrum resources with each other in pro-Par-DF, improper selection of the SCs for them can make the DF ineffective. Fig. 8 shows an example. In Fig. 8(a), the new SC of $C_1$ cuts the available FS-block on $e'$ into two small fragments and blocks the migration of $C_2$. On the other hand, if the new SC of $C_1$ is selected as that in Fig. 8(b), $C_2$ can be migrated successfully. Apparently, the pro-Par-DF instance in Fig. 8(b) is more effective, since it contributes more to the objective function in (11). We analyze the hardness of pro-Par-DF by transforming it from an optimization problem into a decision problem.

*Definition 9:* For an instance that consists of an EON $G(V, E)$, a set of existing connections $\mathbb{C} = \{C_i\}$, and a positive integer $M$, **the problem of pro-Par-DF** is that whether there exists a reconfiguration scheme that can only use MTV-based retuning to achieve $W > M$, where $W$ is calculated with (11)?

*Theorem 2:* The problem of pro-Par-DF is $\mathcal{APX}$-hard.

*Proof:* We prove the $\mathcal{APX}$-hardness of pro-Par-DF by reducing the General Assignment Problem (GAP) [39] to a related pro-Par-DF.

It is known that the decision version of GAP is $\mathcal{APX}$-hard [39]. An instance of GAP includes a pair $(B, S)$, where $B$ is a set of $N$ bins and $S$ is a set of items. The $k$-th bin in $B$ has a capacity of $c_k$, and if we pack item $i$ into bin $k$, it consumes a capacity of $s_{i,k}$ and generates a profit of $w_{i,k}$. The question is that whether there exists a subset $U \subset S$ that has a feasible packing in $B$ and the profit of the packing is larger than $M$?

We build a special case for the problem of pro-Par-DF, and if we can prove that the special case is $\mathcal{APX}$-hard, then the general problem of pro-Par-DF is also $\mathcal{APX}$-hard. In the special case, the EON only has one fiber link $e$ and there are $N$ available FS-blocks (i.e., bins) on $e$. For all the movable connections (i.e., items) on $e$, $\mathbb{C} = \{C_i\}$, pro-Par-DF needs to find a MTV-based retuning scheme to maximize $W$ in (11). Hence, the special case mimics GAP. However, in GAP, each item $i \in S$ contributes a fixed profit $w_{i,k}$ as long as it is placed in bin $k$, while in the pro-Par-DF, the weight contribution from each $C_i$ (i.e., $w_{i,k}$) is variable and depends on the actual location of the new SC within an FS-block.[2] Therefore, GAP becomes a special case of the constructed Pro-Par-DF, and a solution to the special case of pro-Par-DF will imply a solution to GAP. Since GAP is $\mathcal{APX}$-hard, the special case of pro-Par-DF is also $\mathcal{APX}$-hard. Then, we prove that the problem of pro-Par-DF is $\mathcal{APX}$-hard.[3] ■

### C. Problem Decomposition

In pro-Par-DF, since not all the movable connections are mutually correlated, we can decompose the overall problem into several independent subproblems. As the subproblems can be solved independently, we can process them in parallel and reduce the computational complexity effectively.

The total number of SCs in a given pro-Par-DF problem is $\mathcal{A} = \sum_i |A_i|$, which is bounded by $|\mathbb{C}| \cdot F$. Then, we build a conflict graph (CG) with $\mathcal{A}$ nodes, and each node represents an SC. If two SCs $a_{i,k} \in A_i$ and $a_{m,n} \in A_m$ satisfy the following condition, we connect their nodes in the CG,

$$p_i \cap p_m \neq \emptyset \quad and \quad \sum_{j=1}^{F} a_{i,k}[j] \cdot a_{m,n}[j] \neq 0. \tag{15}$$

Since one and only one SC can be allocated to a connection $C_i$, all the available SCs for $C_i$ are mutually connected and form a complete subgraph in the CG. Fig. 9 shows an example of the CG, where none of the connections in $\{C_1, C_2, C_3\}$ is correlated with any one in $\{C_4, C_5, C_6, C_7\}$. Then, we can decompose the overall pro-Par-DF into two independent subproblems that each processes one of the connection sets.

[2]Note that when the size of an FS-block is larger than the bandwidth requirement of a connection, we can obtain multiple SCs within the FS-block. For instance, FS-block $I_2$ in Fig. 7 can accommodate $SC_1$ and $SC_3$.

[3]Here, similar to the analysis in Section V-B, we still generalize the problem by assuming that $N$ is unbounded and the bandwidth requirement of each connection is not restricted.

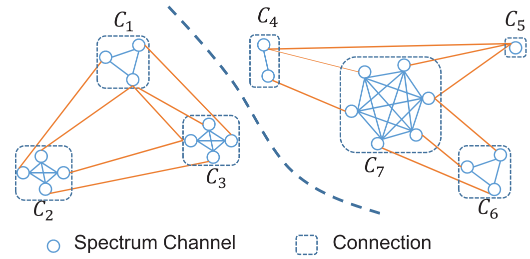



Fig. 9. Decomposition of the conflict graph for pro-Par-DF.

The computational complexity for building the CG is in $O(\mathcal{A}^2)$. We can use the breadth-first-search (BFS) algorithm to find all the desired subgraphs (e.g., the one that includes $\{C_1, C_2, C_3\}$) in the CG. The CG is denoted as $G^{cg}(V^{cg}, E^{cg})$, where $V^{cg}$ and $E^{cg}$ are the node and link sets, respectively. Then, the time complexity of BFS is in $O(|V^{cg}|+|E^{cg}|)$, where $|V^{cg}| = \mathcal{A}$ is bounded by $|\mathbb{C}| \cdot F$ and $|E^{cg}| \in [|V^{cg}|, |V^{cg}|^2]$.

## VII. Efficient Heuristic for Proactive Parallel Defragmentation

Since the pro-Par-DF in EONs is an $\mathcal{APX}$-hard problem and there only exists a 2-approximation [39], we try to obtain a near-optimal solution within a reasonable period of time. As Lagrangian relaxation (LR) can be used to solve complicated optimization problems in optical networks efficiently [40], [41], this section leverages the LR approach to solve the problem of pro-Par-DF based on the ILP formulation in Section VI-A.

### A. Lagrangian Relaxation

The objective function in (11) determines that pro-Par-DF is essentially a maximization problem. Hence, by dualizing the hard-side constraints of the original problem, we can generate an LR problem whose optimal solution provides an upper-bound on the optimal solution of the original one. The LR problem can be solved easily and based on its solutions, we can also construct a feasible solution for the original problem. Since the original problem (i.e., pro-Par-DF) is a maximization problem, the feasible solution provides a lower-bound on its optimal solution. Then, to optimize the solution of the original problem, we apply the aforementioned procedure iteratively. Because of the availability of the lower- and upper-bounds, the LR approach always lets us know the proximity bound of the current feasible solution to the optimal one.

### B. Lagrangian Dual Problem

By dualizing the constraint in (13), we obtain the following dual problem based on the objective in (11)

$$\begin{aligned} &Minimize \;\; Z_{dual}(\Lambda) \\ &= \max_{\{x_{i,k}\}} \left[ \left( \sum_{i=1}^{|\mathbb{C}|} \sum_{k=1}^{|A_i|} w_{i,k} \cdot x_{i,k} \right) + \sum_{e \in E} \sum_{j=1}^{F} \lambda_{e,j} \right. \\ &\qquad \left. \times \left( 1 - \sum_{\{i: C_i \in \mathbb{C}_e\}} \sum_{k=1}^{|A_i|} a_{i,k}[j] \cdot x_{i,k} \right) \right], \\ &\qquad s.t. \quad Eqs.\ (12)\ and\ (14), \end{aligned} \tag{16}$$

where $\Lambda = \{\lambda_{e,k}\}$ is the vector of Lagrangian multipliers. Here, in (16), we have $\Lambda \succeq 0$, which is a necessary condition to guarantee that $Z_{dual}(\Lambda)$ is always an upper-bound on the optimal solution of pro-Par-DF. To obtain $Z_{dual}(\Lambda)$ for any specific $\Lambda$, we regroup relevant terms and get

$$Z_{dual}(\Lambda) = \max_{\{x_{i,k}\}} \left( \sum_{i=1}^{|\mathbb{C}|} \sum_{k=1}^{|A_i|} \overline{w_{i,k}} \cdot x_{i,k} + \sum_{e \in E} \sum_{j=1}^{F} \lambda_{e,j} \right), \tag{17}$$

where $\overline{w_{i,k}}$ is the Lagrangian-modified weight:

$$\overline{w_{i,k}} = w_{i,k} - \sum_{e \in p_i} \sum_{j=1}^{F} \lambda_{e,j} \cdot a_{i,k}[j]. \tag{18}$$

Since the second term in (17) is irrelevant to the decision variables $\{x_{i,k}\}$, we have

$$Z_{dual}(\Lambda) = \max_{\{x_{i,k}\}} \left( \sum_{i=1}^{|\mathbb{C}|} \sum_{k=1}^{|A_i|} \overline{w_{i,k}} \cdot x_{i,k} \right) + \sum_{e \in E} \sum_{j=1}^{F} \lambda_{e,j}. \tag{19}$$

Then, the LR problem for pro-Par-DF is to obtain $Z_{dual}(\Lambda)$ with (19), which can be solved by looking into the following optimization problem

$$\begin{aligned} &Maximize \quad \sum_{i=1}^{|\mathbb{C}|} \sum_{k=1}^{|A_i|} \overline{w_{i,k}} \cdot x_{i,k}, \\ &\quad s.t. \quad Eqs.\ (12)\ and\ (14). \end{aligned} \tag{20}$$

Note that the optimization problem in (20) can be further decomposed into $|\mathbb{C}|$ subproblems, each of which corresponds to a connection $C_i \in \mathbb{C}$ as

$$\begin{aligned} &Maximize \quad \sum_{k=1}^{|A_i|} \overline{w_{i,k}} \cdot x_{i,k}, \\ &\quad s.t. \quad Eqs.\ (12)\ and\ (14). \end{aligned} \tag{21}$$

As these subproblems are independent, they can be solved in parallel. We solve each subproblem by selecting the SC with the maximum weight $\overline{w_{i,k}}$ for connection $C_i$. The computational complexity for calculating $\overline{w_{i,k}}$ is in $O(h_i \cdot F)$, where $h_i$ is the hop-count of the connection's path $p_i$. Then, the complexity for solving each subproblem is in $O(h_i \cdot F \cdot |A_i|)$.

### C. Feasible Solution Construction

By solving the optimization problem defined in (20), we get the solution on $\{x_{i,k}\}$ that corresponds to a specific $\Lambda$, i.e., the set of $x_{i,k}$ that can provide the right value of $Z_{dual}(\Lambda)$ for a fixed $\Lambda$. However, since the constraint in (13) is relaxed by $\Lambda$, the solution on $\{x_{i,k}\}$ might not be a feasible one to the original pro-Par-DF problem. Therefore, we design an efficient algorithm to transform the solution into a feasible one. Meanwhile, we make sure that the dual value $Z_{dual}(\Lambda)$ would not be decreased too much. *Algorithm 2* shows the detailed procedure, in which we allocate the SCs according to the modified Lagrangian weights $\{\overline{w_{i,k}}\}$, while calculating the objective function in (11) with the weights $\{w_{i,k}\}$. The complexity of *Algorithm 2* is in $O(|\mathbb{C}| \cdot \mathcal{A})$.

**Algorithm 2** Feasible Solution Construction for pro-Par-DF

```
C_t = C, W̄ = {w̄_{i,k}}, Z* = 0, {x_{i,k} = 0};
while C_t ≠ ∅ do
    find the maximum w̄_{i,k} in W̄;
    find the corresponding connection C_i ∈ C_t for w̄_{i,k};
    assign the SC a_{i,k} indicated by w̄_{i,k} to C_i;
    mark the corresponding x_{i,k} as 1;
    C_t = C_t \ {C_i};
    find all the SCs that overlap with the assigned one in
    the spectrum domain and set their w̄_{i,k} as −∞;
    Z* = Z* + w_{i,k};
end while
return (Z*);
```

### D. Solving Lagrangian Dual Problem

Since $Z_{dual}(\Lambda)$ is a piecewise linear function and thus cannot be differentiable everywhere, we use the sub-gradient method [42] to solve the Lagrangian dual problem. Specifically, we compute a sequence of $\Lambda$ in iterations such that $Z_{dual}(\Lambda)$ converges to the minimum. We set an initial value to $\Lambda$ (e.g., $\Lambda = \mathbf{0}$) and in each iteration, we update it as

$$\Lambda_{n+1} = \Lambda_n - \mu_n \cdot \mathbf{f}(\Lambda_n), \tag{22}$$

where $\Lambda_n$ and $\mu_n$ are the Lagrangian multiplier and step-size obtained in the $n$-th iteration, respectively. The vector $\mathbf{f}(\Lambda)$ is the sub-gradient of $Z_{dual}(\Lambda)$ with respect to $\Lambda$, i.e.,

$$\mathbf{f}(\Lambda) = \frac{\partial Z_{dual}}{\partial \Lambda}, \tag{23}$$

and the elements in $\mathbf{f}(\Lambda)$ are as follows

$$f_{e,j} = 1 - \sum_{\{i: C_i \in \mathbb{C}_e\}} \sum_{k=1}^{|A_i|} a_{i,k}[j] \cdot x_{i,k}. \tag{24}$$

The choice of $\mu_n$ can have a relatively large effect on the algorithm's convergence performance, and the one that works well in practice is [42]

$$\mu_n = \frac{\nu \cdot (Z_{dual}(\Lambda_n) - Z^*)}{\|\mathbf{f}(\Lambda_n)\|^2}, \tag{25}$$

where $Z_{dual}(\Lambda_n)$ is obtained by applying the processing in Section VII-B and solving the optimization problem in (20) for the Lagrangian multiplier $\Lambda_n$, $Z^*$ is the best-known feasible solution to the original pro-Par-DF problem, which is got by using the procedure in Section VII-C, and $\nu$ is a scalar whose initial value is $\nu = 2$. Note that for the sub-gradient method, the value of $\nu$ will be divided by 2 every time when $Z_{dual}(\Lambda_n)$ fails to improve after a fixed number of iterations.

In certain cases, the condition $\Lambda \succeq 0$ may be violated when using the aforementioned scheme. When this happens, we will project the solution back to the feasible region, which can be done by using the following formula

$$(\Lambda_{n+1})_{e,j} = \max\left\{0,\ (\Lambda_n - \mu_n \cdot \mathbf{f}(\Lambda_n))_{e,j}\right\}. \tag{26}$$

In (16), a large number of inequalities are dualized. Here, we refer to those inequalities that directly contribute to the optimization problem in (16) as active inequalities, since they play important roles on deciding $\Lambda$. Specifically, an active inequality satisfies either $f_{e,j} < 0$ or $\lambda_{e,j} > 0$. As the percentage of active inequalities is relatively small in our problem, $\mu$ decreases very

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**Algorithm 3** Overall LR-pro-Par-DF Algorithm

1: $n = 1$, $\Lambda_n = \mathbf{0}$, $\nu = 2$;
2: $m = 0$, $ub = +\infty$, $lb = 0$;
3: **while** the stopping-condition is not satisfied **do**
4: calculate $\{\overline{w_{i,k}}\}$ with $\Lambda_n$ and $\{w_{i,k}\}$;
$\{$*Solve the LR problem to update the upper-bound*$\}$
5: **for** each connection $C_i \in \mathbb{C}$ **do**
6: solve its subproblem defined in (21);
7: **end for**
8: calculate $Z_{dual}(\Lambda_n)$ with (17);
9: **if** $Z_{dual}(\Lambda_n) < ub$ **then**
10: $ub = Z_{dual}(\Lambda_n)$, $m = 0$;
11: **else if** $m > Th$ **then**
12: $\nu = \nu/2$, $m = 0$;
13: **else**
14: $m = m + 1$;
15: **end if**
$\{$*Build a feasible solution to update the lower-bound*$\}$
16: invoke *Algorithm 2* to construct a feasible solution with $Z_{dual}(\Lambda_n)$ and obtain $Z^*$;
17: **if** $Z^* > lb$ **then**
18: $lb = Z^*$;
19: **end if**
$\{$*Update the parameters with sub-gradient method*$\}$
20: update $\Lambda_n$ and $\mu_n$ with (22)–(26);
21: $n = n + 1$;
22: **end while**

fast in iterations. To overcome this, we implement the strategies developed in [43], i.e., only apply $\{f_{e,j}\}$ from the active inequalities in (25) for updating $\mu$.

### *E. Overall Algorithm Procedure*

The overall procedure for solving the problem of pro-Par-DF with LR (LR-pro-Par-DF) is shown in *Algorithm 3*, whose key part consists of three phases. The first phase covers *Lines* 5–15, where we solve the LR problem for a specific $\Lambda = \Lambda_n$ by solving $|\mathbb{C}|$ independent subproblems and obtain the optimal solution $Z_{dual}(\Lambda_n)$. According to the theory of LR, $Z_{dual}(\Lambda_n)$ provides an upper-bound on the solution of the original pro-Par-DF problem. If the upper-bound has not been updated for a fixed number of iterations (denoted as $Th$), $\nu$ is divided by 2. *Lines* 16–19 represents the second phase, where we construct a feasible solution to the original pro-Par-DF problem, if the solution from the Lagrangian dual problem is not a feasible one. Then, the feasible solution $Z^*$ provides a lower-bound on the solution of the original pro-Par-DF problem. *Lines* 20–21 are for the third phase, where we update the parameters, i.e., the Lagrangian multipliers and the step-size, using the sub-gradient method described in Section VII-D. The three phases will be repeated in iterations until we find a reasonably good feasible solution to the original pro-Par-DF problem.

## VIII. Performance Evaluation and Discussion

In this section, we describe the numerical simulations for performance evaluation and discuss the simulation results.

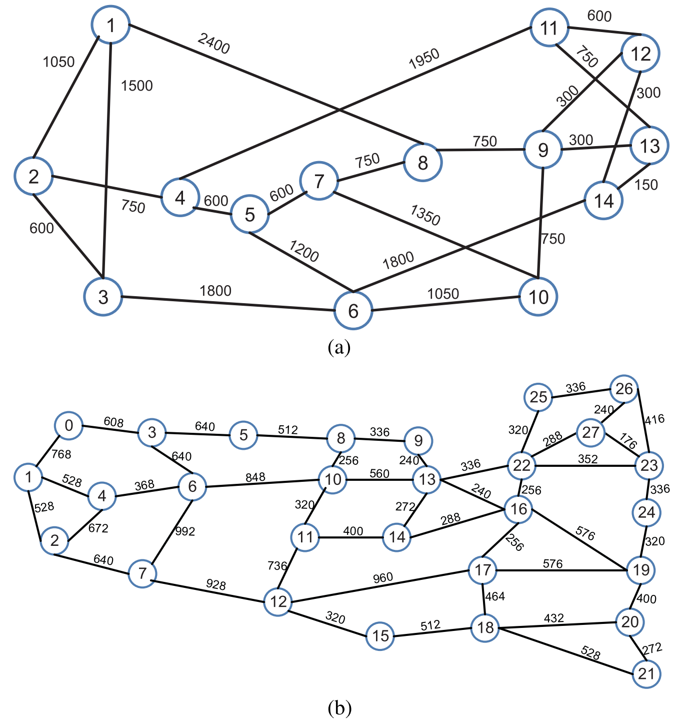
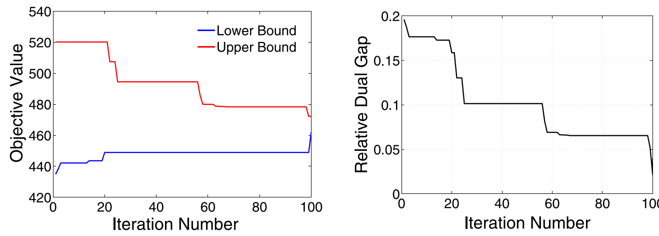


Fig. 10. EON topologies in simulations (link lengths in kilometers). (a) 14-node NSFNET topology. (b) 28-node USB topology.

### *A. Simulation Setup*

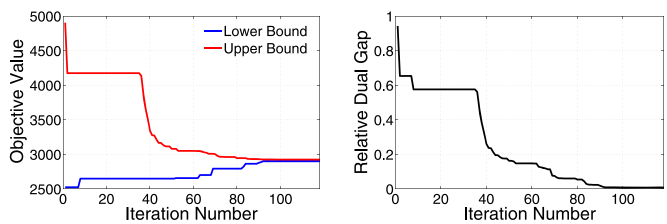
In order to evaluate the performance of pro-Par-DF algorithms, we design simulations with the 14-node NSFNET and 28-node USB topologies that are shown in Figs. 10(a) and 10(b), respectively. We assume that the bandwidth of each FS is 12.5 GHz and each fiber link can accommodate $F = 358$ FS', which correspond to a total bandwidth of 4.475 THz for the C-band. The dynamic connection requests are generated with the Poisson traffic model that their arrivals following the Poisson process with an average rate of $\beta$ per time-unit, while their lifetime follows the negative exponential distribution with an average of $1/\gamma$ time-units. Hence, the traffic load can be quantified with $\beta/\gamma$ in Erlangs. The source and destination nodes of each request are randomly selected, and the bandwidth requirement is uniformly distributed within [1,16] and [1,10] FS' for NSFNET and USB, respectively. At each traffic load, we obtain the performance metrics by serving 20,000 requests on average to guarantee sufficient statistical accuracy.

When a request comes in initially, it is served with the $K$-shortest-path RSA algorithm (KSP) [19], in which we precalculate $K$ shortest paths for the source-destination pair and select the shortest one that has enough spectrum resource to accommodate the request, and then the first-fit spectrum allocation is used. During dynamic network operation, when the EON's spectra become fragmented, we apply a Par-DF algorithm to retune certain movable connections for DF. We utilize the five metrics defined in Section III-D, i.e., the BBP, Network Utilization, DF Disruption, DF Latency and DF cost, to evaluate the DF algorithms. The simulation environment is MATLAB 2012b running on a personal computer with 3.10 GHz Intel Core i3-2100 CPU and 8 GB RAM.



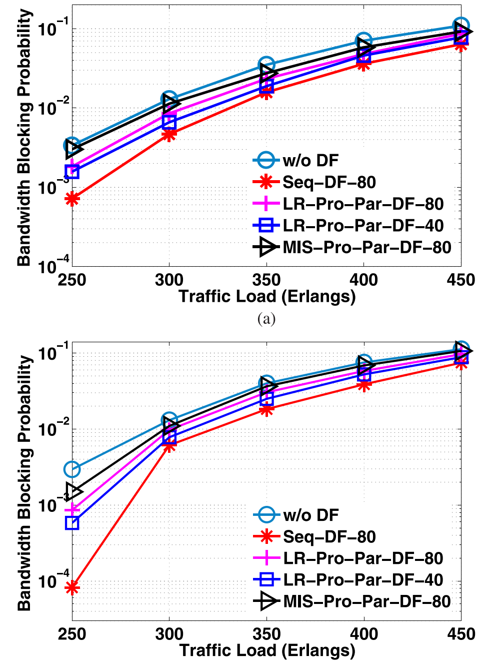

(a) (b)

Fig. 11. Convergence performance of LR-pro-Par-DF (NSFNET topology). (a) Lower- and upper-bounds; (b) relative dual gap.




(a) (b)

Fig. 12. Convergence performance of LR-pro-Par-DF (USB topology). (a) Lower- and upper-bounds; (b) relative dual gap.




(a)

(b)

Fig. 13. Results on BBP. (a) NSFNET topology; (b) USB topology.

TABLE I
AVERAGE COMPUTATION TIME PER DF OPERATION (SECONDS)

| Traffic Load (Erlangs) | LR-pro-Par-DF | | ILP | |
|---|---|---|---|---|
| | NSFNET | USB | NSFNET | USB |
| 250 | 2.1852 | 3.9611 | 19.1112 | 58.2013 |
| 300 | 2.5200 | 4.0172 | 20.3976 | 79.1808 |
| 350 | 2.8605 | 5.8646 | 21.1823 | 86.3327 |
| 400 | 3.4252 | 6.7741 | 23.6836 | 88.9920 |
| 450 | 3.6739 | 6.7748 | 24.4369 | 93.4161 |

### B. Benchmark Algorithms

We use the following two DF algorithms as benchmarks.

- *Seq-DF*: Sequential DF in which path rerouting is not allowed for connection reconfiguration, and the spectrum retuning is done with a greenfield scheme, i.e., DF can retune an existing connection to any spectral location on its path, as long as a feasible reconfiguration sequence can be obtained. Specifically, we sort all the existing connections based on their bandwidth requirements in descending order, calculate new spectrum assignment for each of them sequentially, and then perform traffic migration with the auxiliary graph based approach discussed in Section IV-A and [19].
- *MIS-pro-Par-DF*: Maximal independent set (MIS) based pro-Par-DF algorithm tries to retune as many existing connections to lower spectral locations as possible. It builds a conflict graph (CG) with the method discussed in Section VI-C, and then finds the MIS in each independent subgraph of CG for DF. Note that an MIS indicates not only the maximal connection set for parallel migration but also the target spectral locations.

The simulations trigger a DF operation when a fixed number (i.e., $N$) of connections have expired in the EON, and include $N$ in the notations of the algorithms. For instance, "Seq-DF-80" means that the Seq-DF algorithm triggers a DF operation every time when 80 connections have expired in the EON.

### C. Simulation Results

*1) Convergence Performance:* We first evaluate the convergence performance of LR-pro-Par-DF. Figs. 11(a) and 12(a) show the convergence of the algorithm for one DF operation, in NSFNET and USB topologies, respectively, when the network status is randomly selected with the traffic load at 400 Erlangs. The simulations have $Th = 25$. We observe that for both cases, the lower- and upper-bounds on the optimal solution converge effectively along with the iterations. We define the relative dual gap (RDG) as follows to quantify the convergence performance [41]

$$RDG = \frac{ub - lb}{lb}, \tag{27}$$

where $ub$ and $lb$ are the upper- and lower-bounds obtained in each iteration. RDG provides the bound of the real dual gap, i.e., the gap between the optimal and current solutions. We plot the simulation results on RDG in Figs. 11(b) and 12(b), in which we can see that the value of RDG becomes less than 0.05 within 100 and 150 iterations, for the simulations with the NSFNET and USB topologies, respectively. Moreover, we also verify that in most of the simulations, RDG can converge below 0.05 within 500 iterations, no matter what traffic load the EON carries. The results on average computation time per DF are summarized in Table I. We also use CPLEX to solve the ILP for pro-Par-DF in Section VI-A and record the computation time in Table I. The results indicate that compared with the ILP, LR-pro-Par-DF can reduce the computation time for almost one magnitude.

*2) Bandwidth Blocking Probability (BBP):* Fig. 13 shows the simulation results on BBP, which indicate that compared with the one without DF, all the DF algorithms reduce BBP, especially when the traffic load is low. The benefit of DF becomes less when the traffic load increases. This is because when the traffic load is higher, the network becomes more crowded and the margin for connection reconfiguration is less.

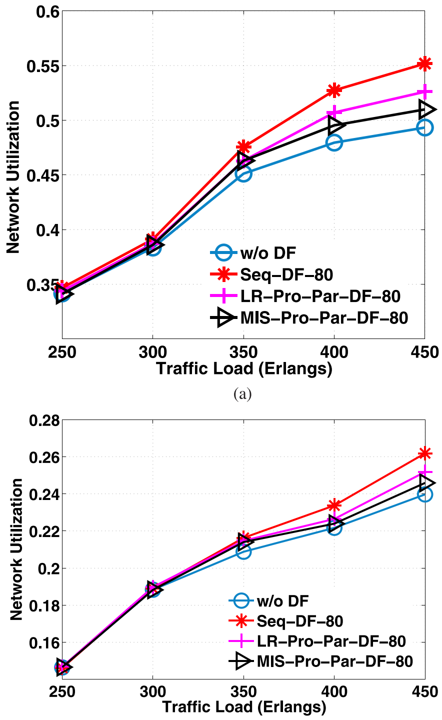



Fig. 14. Results on Network Utilization. (a) NSFNET topology; (b) USB topology.

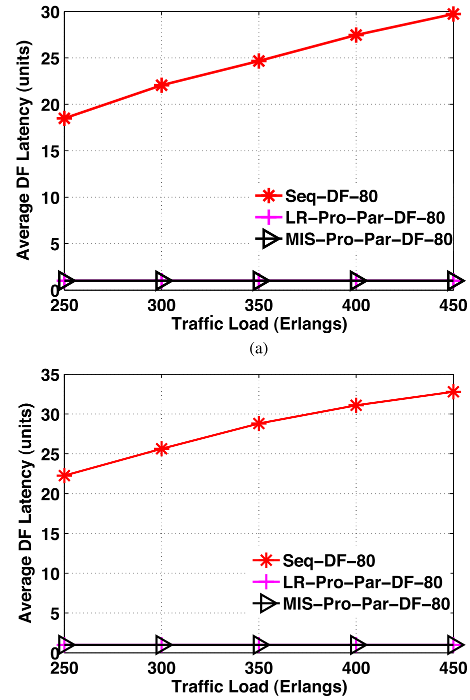



Fig. 15. Results on average DF Latency. (a) NSFNET topology; (b) USB topology.

Among all the DF algorithms, MIS-pro-Par-DF-80 provides the worst BBP performance, and this suggests that by only maximizing the number of connections to reconfigure, we may not achieve the most effective spectrum consolidation in DF. In the simulations, LR-pro-Par-DF-80 achieves better BBP performance than MIS-pro-Par-DF-80. The reasoning behind this performance improvement is twofold. First of all, LR-pro-Par-DF processes the critical connections (i.e., those with relatively large weights $w_{i,k}$) with high priority and tries to retune them first. Secondly, LR-pro-Par-DF retunes the movable connections to the right spectral locations (i.e., to the lower spectral end as far as possible). Moreover, we notice that by increasing the frequency of DF operations in LR-pro-Par-DF, we can further improve its BBP performance. For instance, LR-pro-Par-DF-40 outperforms LR-pro-Par-DF-80 in both topologies. Since Seq-DF has more freedom on spectrum retuning, Seq-DF-80 provides better BBP performance than MIS-pro-Par-DF-80 and LR-pro-Par-DF-80. However, as we will show later, Seq-DF performs much worse on DF Disruption, DF Latency and DF cost, due to the intrinsic drawback caused by the sequential scenario.

*3) Network Utilization:* Fig. 14 shows the results on Network Utilization at different traffic loads, from which we can see that for the traffic load within [250, 450] Erlangs, the EON is in the lightly or moderately loaded condition. We also notice that if an algorithm can achieve better BBP performance than another, its Network Utilization is also higher. Besides, when the traffic load is lower than 300 Erlangs, all the algorithms perform similarly on Network Utilization. Hence, at relatively low traffic loads, BBP is the metric that we should use to evaluate the algorithms, and when the traffic load is higher than 350 Erlangs, Network Utilization can tell the difference on the algorithms' performance more clearly. Note that EONs are usually considered for backbone or core networks, which carry huge volumes of traffic. Hence, even a relatively small improvement on spectrum utilization or blocking probability can bring considerable revenue gains for network operators.

*4) DF Latency and Disruption:* Fig. 15 shows the results on the average DF Latency per DF operation. In the simulations, we assume that the average DF Latency per connection retuning (i.e., $L_P$) is one unit. As the pro-Par-DF algorithms process all the movable connections in parallel, their results on the average DF Latency stay as 1 unit for all the traffic loads. On the other hand, we observe that for Seq-DF, the average DF Latency increases with the traffic load because for a higher traffic load, the algorithm needs to process more connections in each DF operation, which leads to a larger-sized auxiliary graph for dependency. Therefore, the average DF Latency increases for the reason discussed in Section IV-C. The results in Fig. 15 indicate that the average DF Latency of Seq-DF is more than 15 times longer than that of pro-Par-DF.

Fig. 16 shows the maximum DF Disruption a connection may experience in Seq-DF at different traffic loads. According to the analysis in Section IV-C, when the auxiliary graph for dependency is cyclic, Seq-DF will cause significantly long DF Disruption. The results in Fig. 16 confirm the analysis and show that the DF Disruption increases with the traffic load.

*5) DF Cost:* Fig. 17 shows the results on the average DF Cost, which is defined as the average number of connection reconfigurations per DF operation. We observe that the average DF Cost from Seq-DF is much higher than those from the pro-Par-DF algorithms, for the reason that it reconfigures more connections. Since the objective of MIS-pro-Par-DF is to retune as many connections as possible, it provides a higher DF Cost than LR-pro-Par-DF. The results in Fig. 17 suggest

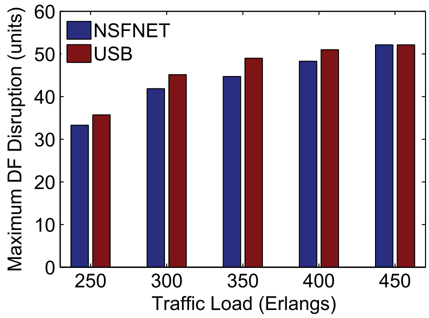



Fig. 16. Results on the maximum DF Disruption in Seq-DF.

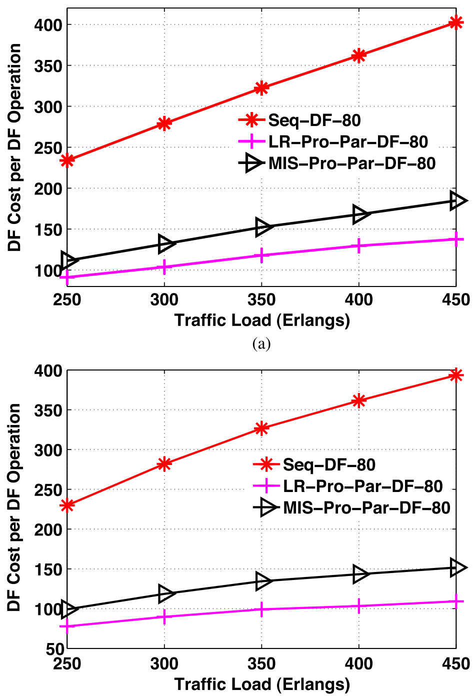



Fig. 17. Results on average DF Cost. (a) NSFNET topology; (b) USB topology.

that LR-pro-Par-DF accomplishes DF in a more cost-effective way, since compared with MIS-pro-Par-DF, it retunes fewer connections and achieves better BBP performance.

From the discussion above, we can see that LR-pro-Par-DF outperforms Seq-DF in terms of the DF Latency, the DF Disruption, and the DF Cost.

## IX. Conclusion and Future Work

This paper investigated the parallelization of DF-related connection reconfigurations in EONs. We first analyzed Par-DF and Seq-DF in dynamic EONs in detail and estimated their performance on the DF Latency and Disruption. Then, we studied two types of Par-DF in EONs, namely, re-Par-DF and pro-Par-DF. We proved that the problem of the former one belongs to the strongly $\mathcal{NP}$-hard class, even though a similar problem in WDM networks can be solved in polynomial time. Subsequently, we investigated the problem of pro-Par-DF and proved that it is an $\mathcal{APX}$-hard problem. In order to obtain a near-optimal solution of pro-Par-DF time-efficiently, we formulated an ILP model and proposed a Lagrangian-relaxation (LR) based heuristic algorithm based on it. Thanks to the LR, we confirmed that the original problem can be decomposed into several independent subproblems, each of which can be solved time-efficiently. Simulation results showed that the LR-based pro-Par-DF algorithm (LR-pro-Par-DF) could provide a near-optimal performance (the relative dual gap is less than 5%) within 500 iterations. The simulations also verified that the proposed LR-pro-Par-DF outperformed Seq-DF in terms of the DF Latency, the DF Disruption and the DF Cost.

Meanwhile, we noticed that there is an overall tradeoff between DF performance and operation cost, i.e., Seq-DF can provide better DF performance but suffers from high operation cost, while Par-DF is just the other way around. However, by triggering Par-DF more frequently, its DF performance can be improved significantly as shown in Fig. 13. Therefore, it would be interesting to study how to approximate the performance of Seq-DF by conducing Par-DF multiple times while keeping the additional operation cost as reasonable. Besides, there are other degrees of flexibilities for DF operations, which have not been addressed in this work, e.g., lightpath rerouting, modulation-level reassignment, etc. Even though considering them in DF may increase the complexity of DF, it would still be promising to investigate them as they might further improve the DF performance. Considering the page limit of the paper, we will study the aforementioned problems in our future work.

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